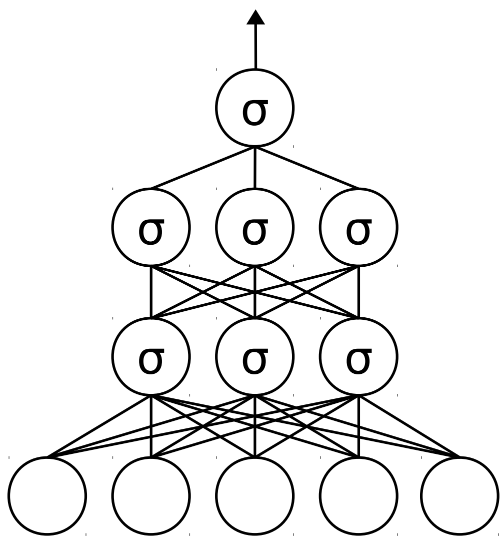
MA2823: Foundations of Machine Learning **Homework 10**

Due December 15, 2017.

Turn in your homework

- as a PDF file
- named HW10_<LastName>_<FirstName>.pdf (no accents)
- at http://tinyurl.com/ma2823-2017-hw

Consider the feed forward neural network depicted below. Although arrows were not depicted, units from one layer are connected unidirectionally to units from the layer above. Each σ depicts a sigmoid unit with a bias; bias units are not represented.



Question 1
How many features is this neural network using?

Solution: 5 input nodes = 5 features.

Question 2
What is the decision function of this neural network?

Solution: Let w_{jhk} denote the weight of the connection from unit j of layer $)k-1)$ to unit h of layer k), and b_{hk} be the bias of unit h of layer k.

- The output of unit h of the first hidden layer is $z_{h1} = \sigma\left(b_{h1} \sum_{j=1}^{5} w_{jh1} x_j\right)$.
- The output of unit t of the second hidden layer is $z_{t2} = \sigma\left(b_{t2} \sum_{h=1}^{3} w_{ht2} z_{h1}\right) = \sigma\left(b_{t2} \sum_{h=1}^{3} w_{ht2} \sigma\left(b_{h1} \sum_{j=1}^{5} w_{jh1} x_j\right)\right)$.

— The output of the network is $f(\boldsymbol{x}) = \sigma\left(b_3 \sum_{t=1}^{3} w_{t3} z_{t2}\right) = \sigma\left(b_3 \sum_{t=1}^{3} w_{t3} \sigma\left(b_{t2} \sum_{h=1}^{3} w_{ht2} \sigma\left(b_{h1} \sum_{j=1}^{5} w_{jh1} x_j\right)\right)\right)$.

Question 3

What error function do you want to consider for this network?

Solution: We will assume this is a classification neural network (the output unit is a sigmoid), and unse the cross-entropy error.

Question 4

What is the update rule for a weight connecting an input unit to a unit of the first hidden layer?

Solution: The update rule is $w_{jh1} \leftarrow w_{jh1} - \eta \frac{\partial \text{Error}(\boldsymbol{x}^i, y^i)}{\partial w_{jh1}}$.

$$\text{Error}(\boldsymbol{x}^i, y^i) = -y^i \log(f(\boldsymbol{x}^i)) + (1 - y^i) \log(1 - f(\boldsymbol{x}^i)).$$

$$\frac{\partial \text{Error}(\boldsymbol{x}^i, y^i)}{\partial w_{jh1}} = \frac{\partial \text{Error}(\boldsymbol{x}^i, y^i)}{\partial f(\boldsymbol{x}^i, y^i)} \frac{\partial f(\boldsymbol{x}^i, y^i)}{\partial z_{t2}} \frac{\partial z_{t2}}{\partial z_{h1}} \frac{\partial z_{h1}}{\partial w_{jh1}}$$

$$\frac{\partial \text{Error}(\boldsymbol{x}^i)}{\partial f(\boldsymbol{x}^i)} = -\frac{y^i}{1 - f(\boldsymbol{x}^i)} - \frac{(1 - y^i)}{(1 - f(\boldsymbol{x}^i))} = \frac{f(\boldsymbol{x}^i) - y}{f(\boldsymbol{x}^i)(1 - f(\boldsymbol{x}^i))}$$

Remembering that $\sigma'(u) = u'\sigma(u)(1 - \sigma(u))$,

$$\frac{\partial f(\boldsymbol{x}^i)}{\partial z_{t2}} = w_{t3} f(\boldsymbol{x}^i)(1 - f(\boldsymbol{x}^i)$$

$$\frac{\partial z_{t2}}{\partial z_{h1}} = w_{ht2} z_{t2}(1 - z_{t2})$$

$$\frac{\partial z_{h1}}{\partial w_{jh1}} = x_j^i z_{h1}(1 - z_{h1})$$

Finally,

$$\frac{\partial \text{Error}(\boldsymbol{x}^i, y^i)}{\partial w_{jh1}} = -w_{t3}(f(\boldsymbol{x}^i) - y^i) w_{ht2} z_{t2}(1 - z_{t2}) x_j^i z_{h1}(1 - z_{h1}).$$